Economic Growth and Welfare Systems

Jean Monnet Chair in
European Integration Studies
Prof. PASQUALE TRIDICO
Università Roma Tre

Neoclassical model of growth (Solow 1956)

Two inputs, K, L
A production function Cobb-Douglas
Y= $F(K,L) = K^{\propto}L^{1-\propto} \quad 0 < \propto < 1$

Constant return to scale (decreasing marginal return for each factor)

$F(hK,hL) = hF(K,L) = hY$
$F(K/L, L/L) = F/L(K/L, L/L) = y = F(k)$

The Cobb-Douglas has constant returns to scale; if you double (halve) the amount of each input, you double (halve) output .

Hypotheses (1)

- Substitutability between K and L
- Variation of different combinations of K and L
- I depends on r
- The hedge knife (Harrod instability) is solved through the variation of $v = K/Y$ allowed for by prices flexibility (prices of factors)

Hypotheses (2)

- no government purchase of goods and services:

$Y = C + I$ for each t

- Hence saving S equals gross investment I

$Y - C = S = I$ for each t

Population growth and labour force

- Hp: the number of workers growth at the same rate of pop growth
- $Lt = L0e^{nt}$
- Population growth and work force growth exhibit exponential growth:
- $\frac{\dot{L}t}{Lt} = n$

Accumulation of capital, S, I and δ

- Output depends on K and L

- $\dot{K} = I - \delta K = sY - \delta K$
- $\dot{K}$ *is the change in capital stock over time*
- Income is equal to output
- Individuals save always the same fraction (with s<1, > 0 of Y)
- Aggregate saving = to gross I

  sY=S=I

K depreciates over time (with $\delta > 0, < 1\ of\ K$)

Accumulation of capital equation (*)

- $Divide\ \dot{K} = I - \delta K = sY - \delta K$ by K
- $\frac{\dot{K}}{K} = s\frac{Y}{K} - \delta\frac{K}{K} = s\frac{Y}{K} - \delta$

Accumulation of capital per workers

$$\frac{K}{L} = k$$

$$if\ \frac{\dot{K}}{K} = \frac{\dot{L}}{L} \rightarrow \frac{\dot{k}}{k} = 0$$

$$if\ \frac{\dot{K}}{K} > \frac{\dot{L}}{L} \rightarrow \frac{\dot{k}}{k} > 0$$

$$if\ \frac{\dot{K}}{K} < \frac{\dot{L}}{L} \rightarrow \frac{\dot{k}}{k} < 0$$

Hypotheses

- perfect competition
- Price takers firms

$$Y = F(K,L) = K^{\alpha} L^{1-\alpha}$$

Profit maximizations solve the following problem

Max F(K,L)$-rK - wL$

K,L

PML=w, PMK=r

$$w = \frac{\partial F}{\partial L} = (1-\alpha)\frac{Y}{L}$$

$$r = \frac{\partial F}{\partial K} = \alpha\frac{Y}{K}$$

the share of Y for Labour is

$$1-\alpha = w\frac{L}{Y}$$

the share of Y for Kapital is

$$\alpha = r\frac{K}{Y}$$

Production function in terms of output per worker

$$y = \frac{Y}{L} = \frac{1}{L}\left(K^{\alpha} L^{1-\alpha}\right) = \left(K^{\alpha} L^{1-\alpha-1}\right) = K^{\alpha} L^{-\alpha} = \frac{K^{\alpha}}{L^{\alpha}} = \left(\frac{K}{L}\right)^{\alpha} = k^{\alpha}$$

$$k = \frac{K}{L}$$

$$\alpha < 1$$

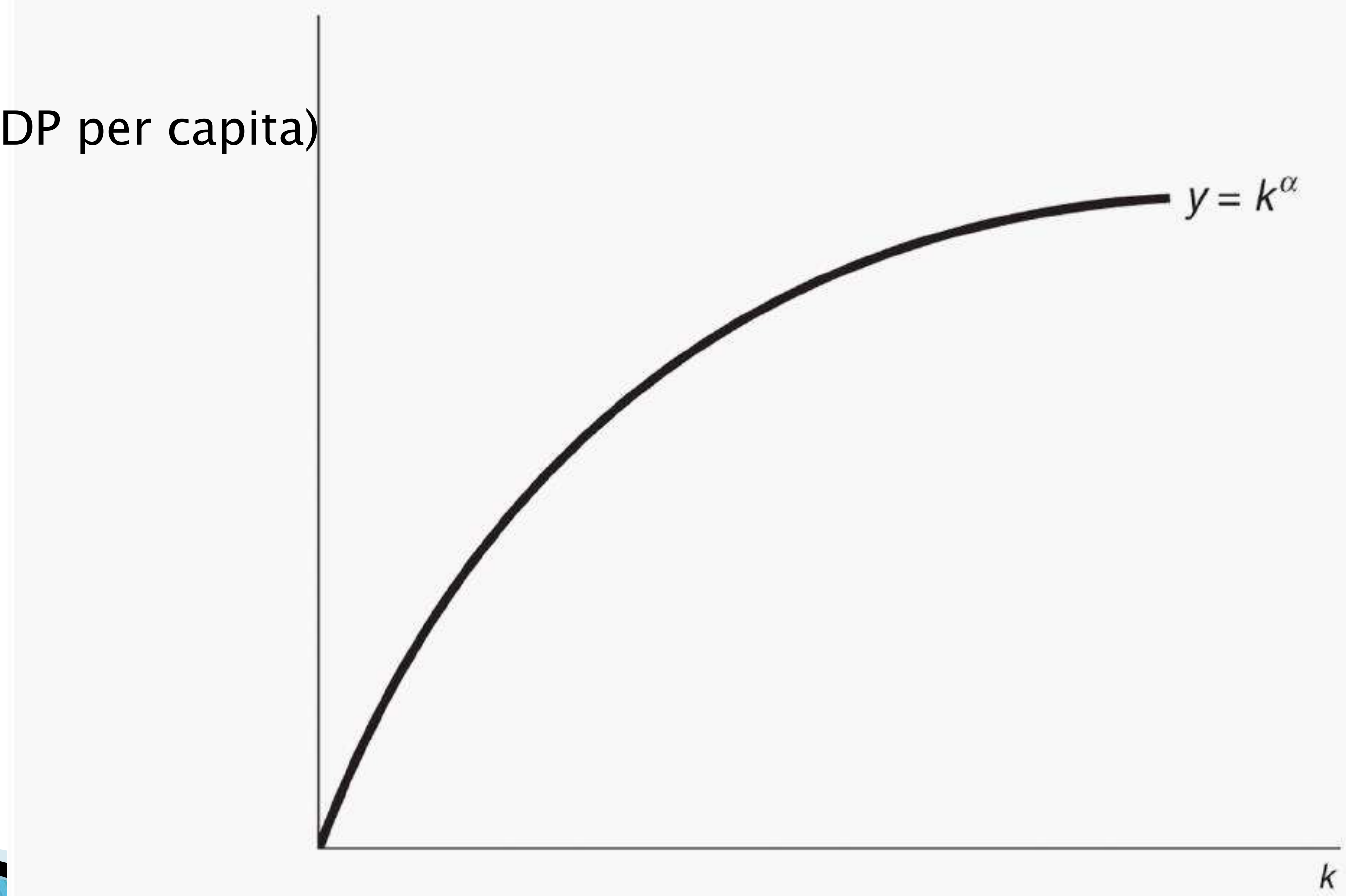
$$y = k^{\alpha}$$

First key equation of Solow

Capital per worker has a diminishing marginal output ($\propto < 1$). If k rises, output per worker rises, but…with decreasing return (less and less Y with more K)

FIGURE 2.1 A COBB-DOUGLAS PRODUCTION FUNCTION

$y = k^{\propto}$ (GDP per capita)



The second fundamental equation

- $\dot{K} = sY - \delta K$ * (capital accumulation equation)
- About how capital accumulates
- The change of capital stock is equal to the amount of gross investment sY less the amount of depreciation δK
- Workers/consumers save a constant fraction s of Y
- δ=5% (for instance)
- $\dot{K} = \frac{dK}{dt}$
  $\dot{K}$ *indicate the derivative with respect to time,*
- *or simply,* $K_{t+1} - K_t$

Log and derivative to get the second fundamental equation

- to study the evolution of output per person we write the capital accumulation equation * per person, so we divide K per L
  - This rewriting is easily and mostly accomplished using Log and then derivative

$$\frac{K}{L} = k$$

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L}$$

$$from* \rightarrow \frac{\dot{K}}{K} = s\frac{Y}{K} - \delta; and \frac{\dot{L}}{L} = n$$

$$\frac{\dot{k}}{k} = s\frac{Y}{K} - \delta - n = s\frac{y}{k} - \delta - n$$

$$\dot{k} = sy - (n+\delta)k$$

the second fundamental equation of Solow

$$\dot{k} = sy - (n + \delta)k$$

- The equation says that change in k per worker is determined by

1. I per workers **sy** which increases k
2. **Depreciation** which decreases K
3. And population growth **n** which decreases k

In each period there are nL new workers. If there were no new investments, capital per worker would decline because of the increase of labour force (by n)

Solving the Solow model

solving a model means to obtain the value of each endogenous var with given values for the exogenous variables

From the two key Solow's equations:

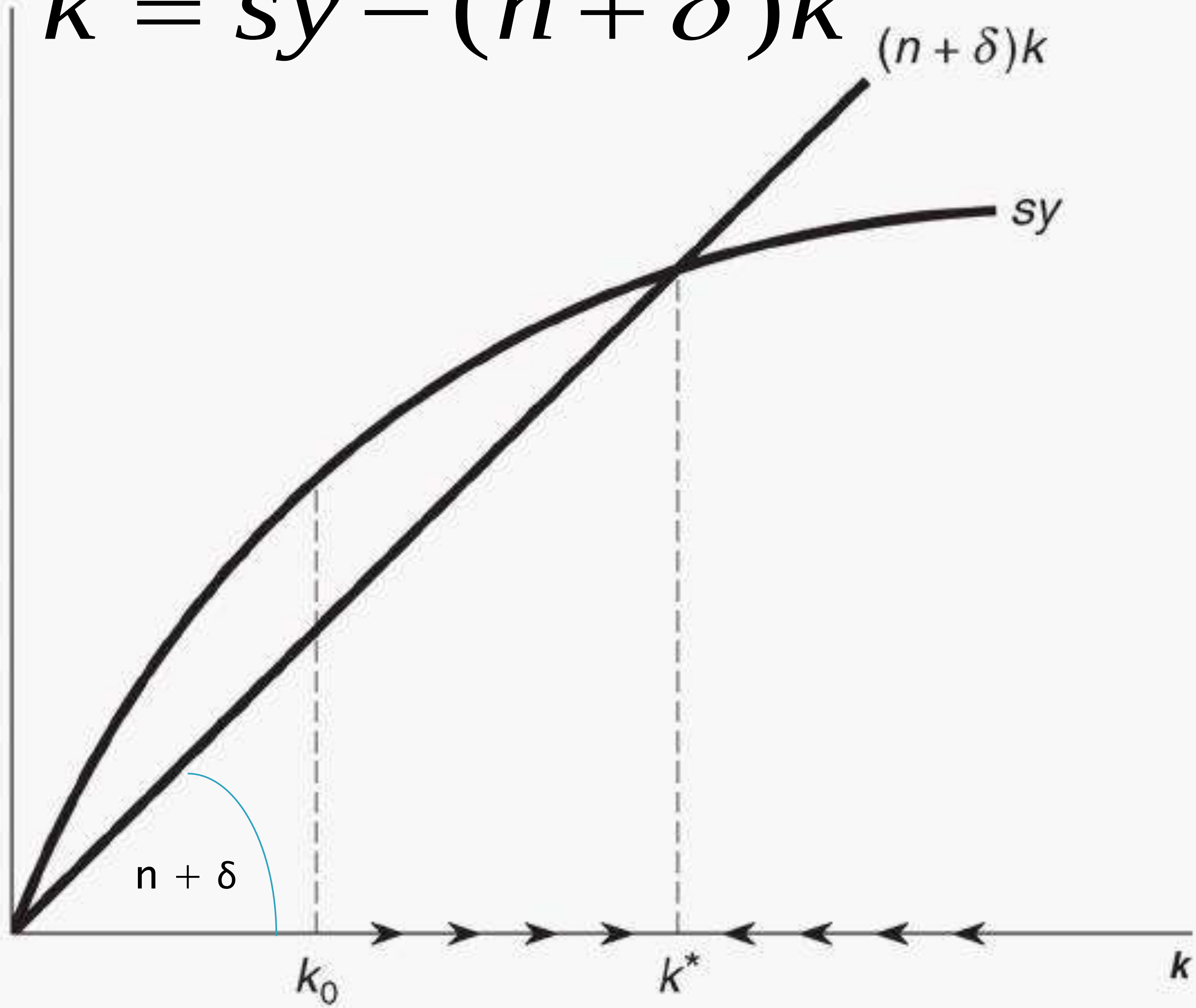
$$y = k^{\alpha}$$

$$\dot{k} = sy - (n + \delta)k$$

How does output per worker evolve over time in this economy? How does in the long run output per worker compare between 2 economies having different rates of investment?

FIGURE 2.2 THE BASIC SOLOW DIAGRAM

$$\dot{k} = sy - (n+\delta)k$$



The Steady State (1) solution

the Solow model predicts that K/L will stabilize at K*
where Investments (sy) are equal to n+d (they offset in K*
pop growth and depreciation). K* is then the Steady state
of the Solow model

$$if : k < k^* \rightarrow sy > (n+\delta)k \rightarrow \dot{k} > 0$$

$$if : k > k^* \rightarrow sy < (n+\delta)k \rightarrow \dot{k} < 0$$

$$(\delta + n)k.$$

is the line where investment per worker is constant
It represents both pop growth n and depreciation, as a fraction of that K

n and K

- For each value of K, n indicates how much of Investment is needed in order to keep K/L constant
- Hence n is the rate of growth g of equilibrium
- The system tends towards $n = g$
- On the left of k^*, I are more of what is necessary in order to keep K/L constant → $K/L > L/L$ (n) → k↑
- On the right of k^*, I are insufficient to keep K/L constant → $K/L < L/L$ (n) →k↓

Solow's model and its solution to Harrod's knife edge

- When $k = k^*$: $sy = (n + \delta)k \longleftrightarrow K/Y = v^*$
- When $k < k^*$: $sy > (n + \delta)k \longleftrightarrow K/Y < v^*$

But in this case, excess supply of capital: r decreases

It becomes more convenient to adopt techniques which use more capital: k↑, K/Y↑ and converges to v*

- When $k > k^*$: $sy < (n + \delta)k \longleftrightarrow K/Y > v^*$

But in this case, excess demand of capital: r increases

It becomes more convenient to adopt techniques which use more labour: k↓, K/Y ↓ and converges to v*

Capital deepening and capital widening

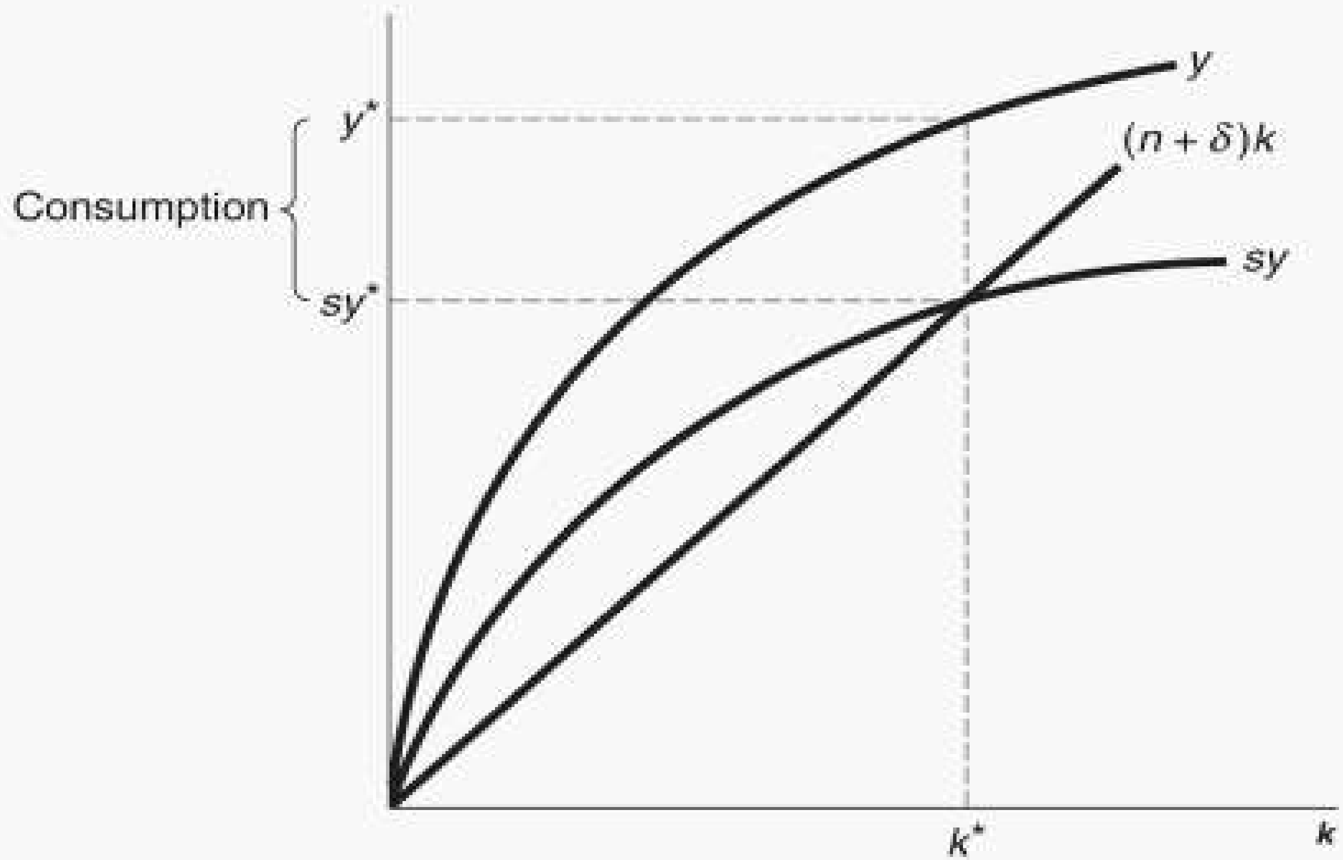
- Between k_0 and before k^* we speak about capital deepening: the capital per worker ratio increases
- When capital per worker ratio is zero (it does not increases) we speak about capital widening (Capital is growing, but because of population growth K/L is constant

The Steady State (2) implications

- This is the solution of the Solow model in general terms.
- If k_0 (the initial level of K per worker), α, s, n and depreciation of capital, are known, one can establishes whether capital per worker will grow or not (so the economy will grow or not).
- This has also very important policy implications as far as policy can influence s (and I), n and the other parameters

Consumption and Saving for Investment

FIGURE 2.3 THE SOLOW DIAGRAM AND THE PRODUCTION FUNCTION



The golden rule

- Is that path of growth that allows for s→K which represents the minimum share of total outcome and also the max possible consumption, or in another way: is the rate of savings which maximizes steady state level of growth and consumption

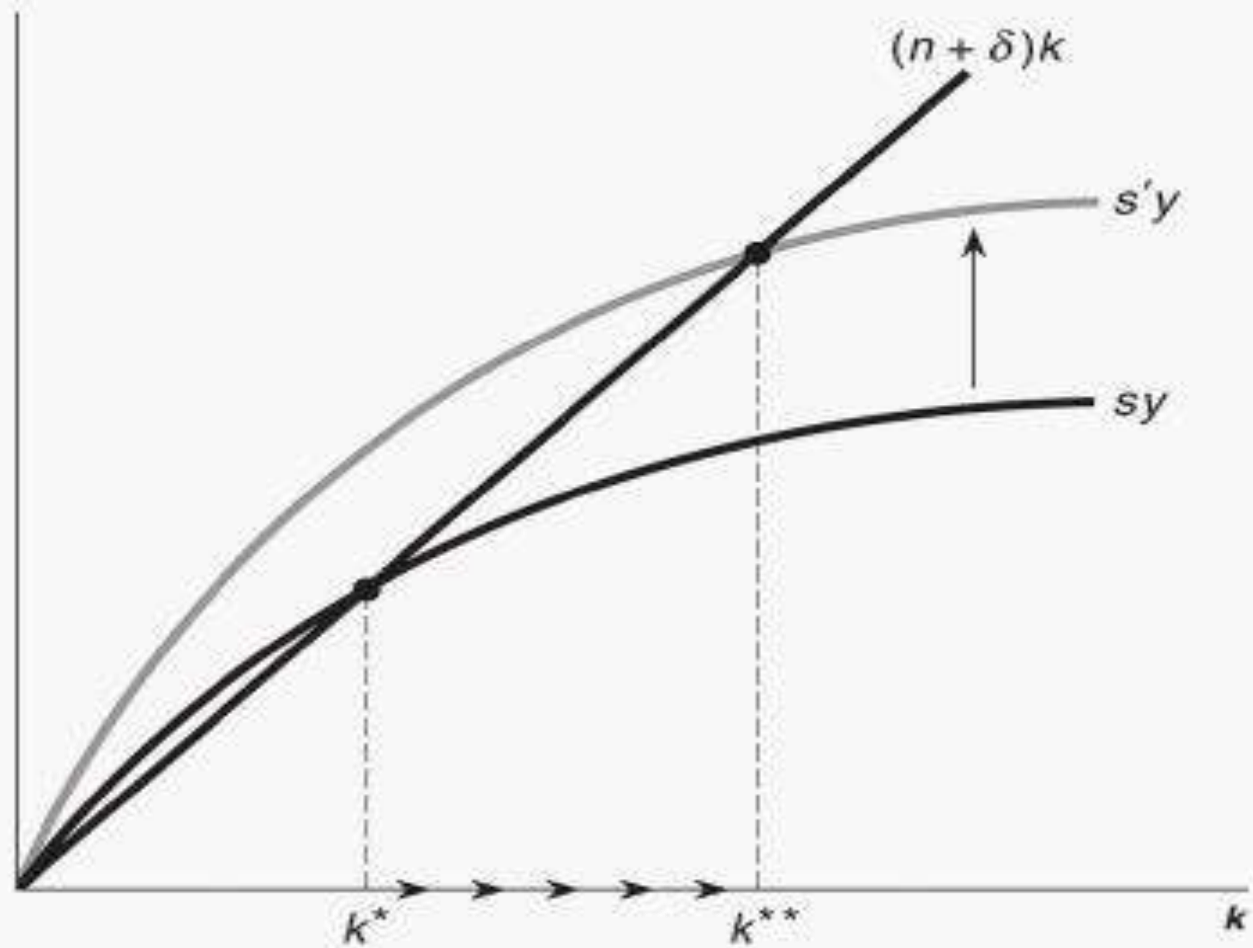
The effect of an increase in s

- The SS rises from k* to k** and the economy will be richer
- Before the new capital/worker ratio capital deepening occurs, and output per worker grows untill the economy reaches the new (higher) rate of steady state
- In k** income will be higher

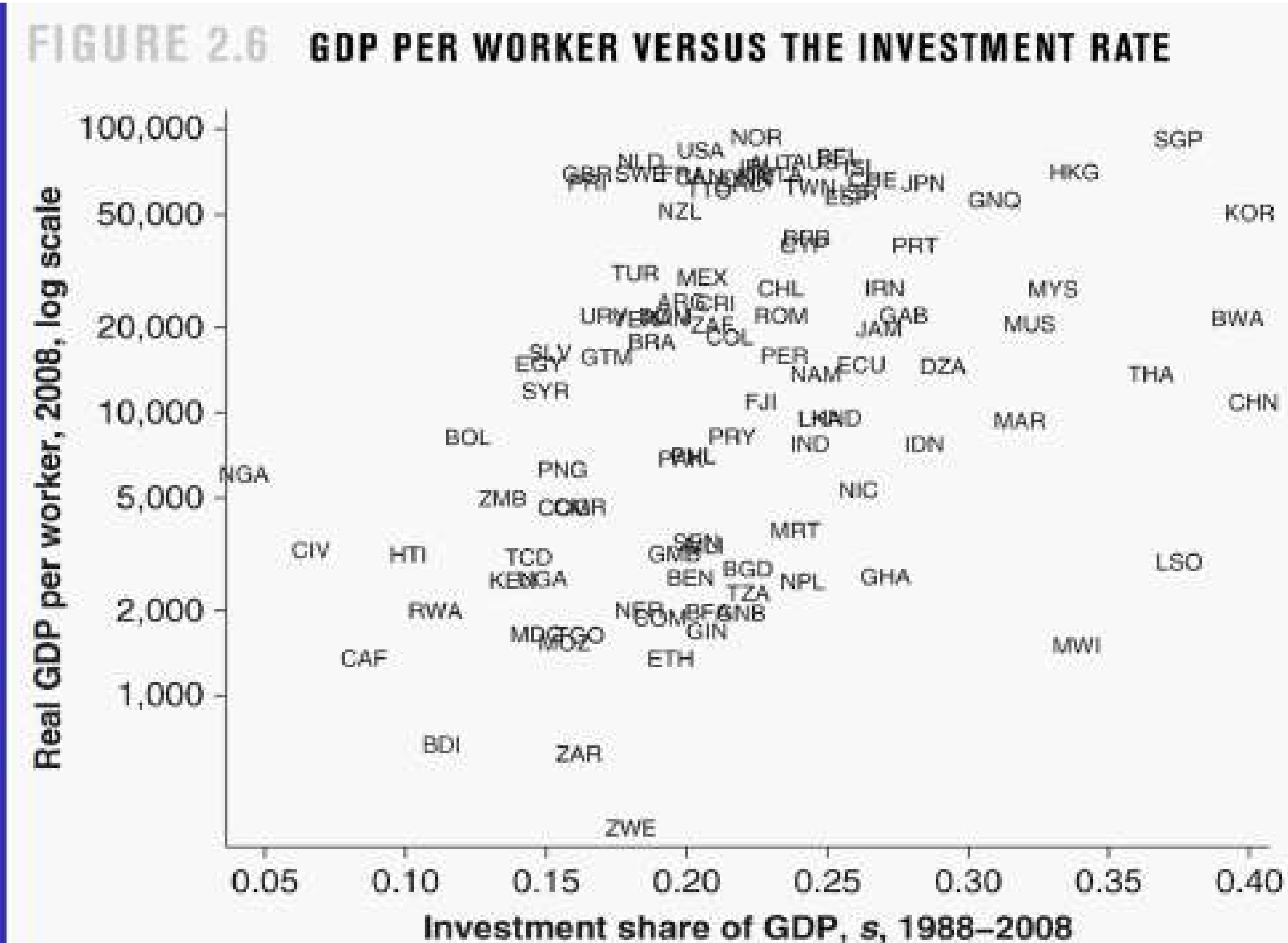
- At the current value of capital stock k* investment per worker **now (after the increase of s)** exceeds the ammount required to keep the capital labour ratio constant and therefore a process of capital deepening restarts and the economy grows (GDP per capita grows) untill when sy reaches the new steady states in K** and is equal to n+d line

If s changes, so that s'>s, the SS is reached at higher rate, growth is faster

FIGURE 2.4 AN INCREASE IN THE INVESTMENT RATE



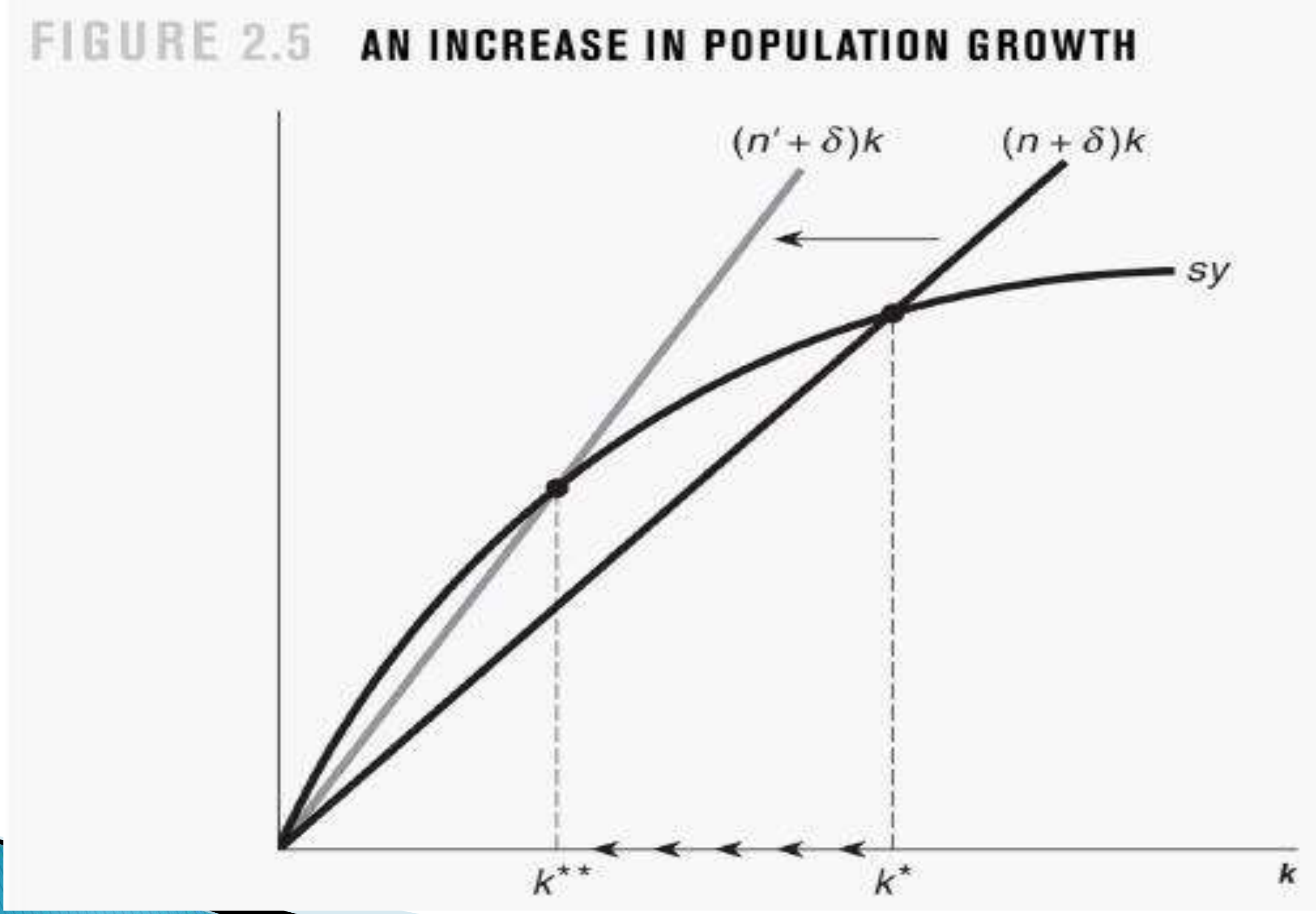
Investment → GDP (+)



The effect of an increase in n

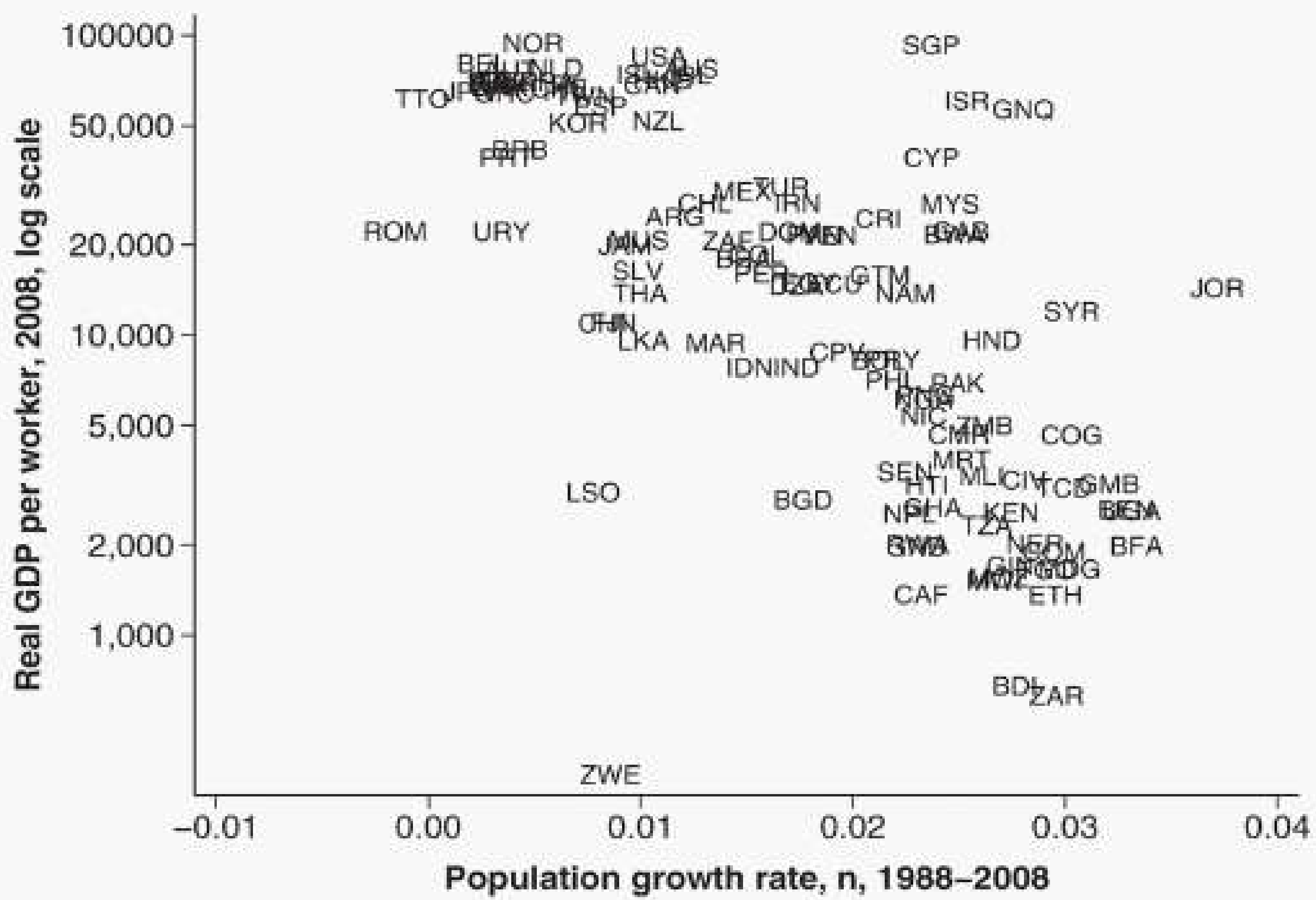
- At the current value of capital stock k* investment per worker **is now (after the increase of n)** no longer high enough to keep the capital labour ratio constant in the face of a rising in the population.
- Therefore the capital labour ratio begins to fall untill the point at which investment (sy) is equal to the new n+depreciation in k**
- At this point the economy has less capital per worker than before and is therefore poorer.
- Per capita output (and icome) is lower after the increase of n
- The SS decreases from k* to k** and the economy will be poorer

The effect of an increase in pop growth



Pop. growth and GDP (–)

FIGURE 2.7 GDP PER WORKER VERSUS POPULATION GROWTH RATES



A trap-case (in LCDs)

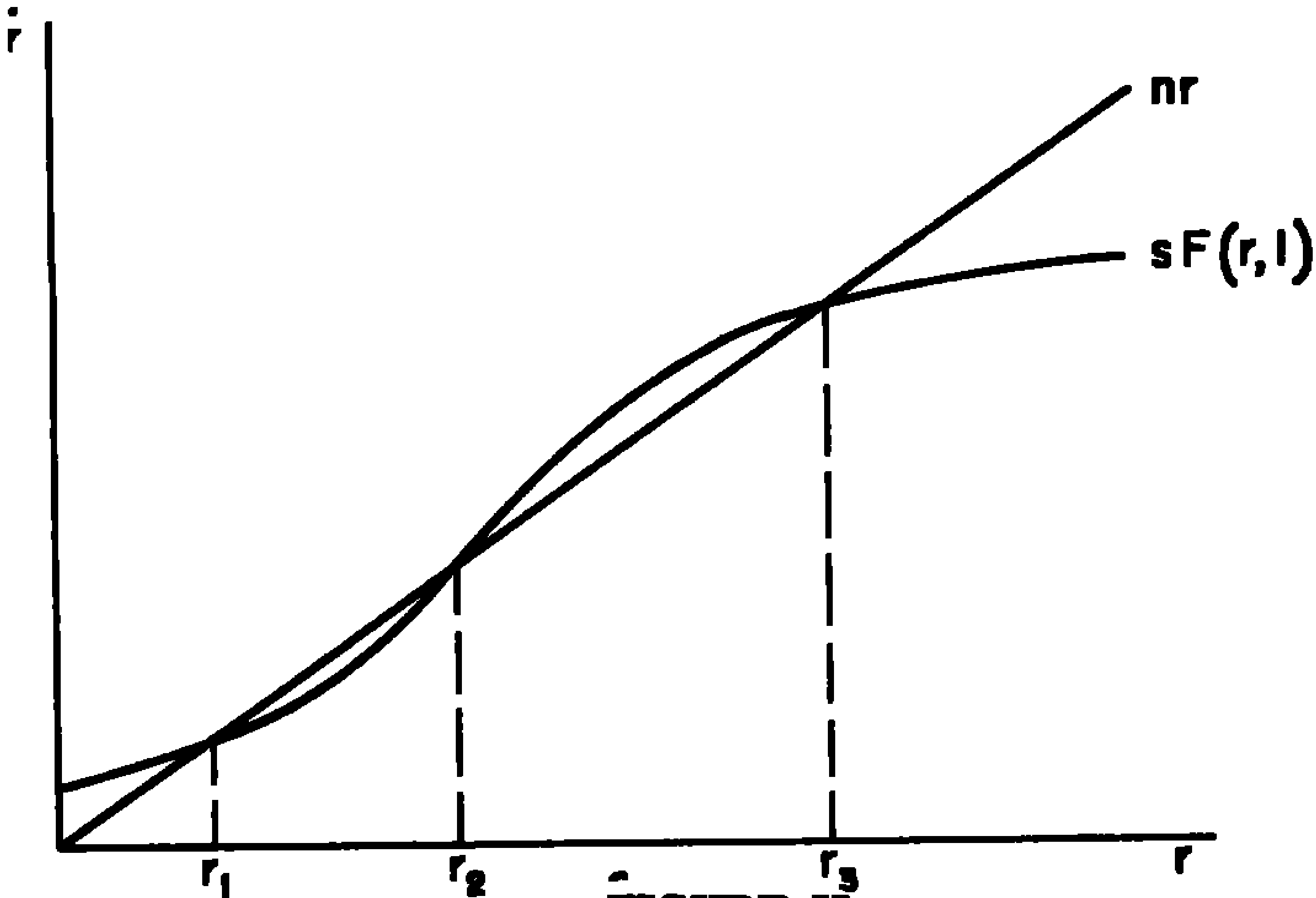


FIGURE II