# Economic Growth and Welfare Systems

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# Models of Economic growth
## (that we will study)

1. Harrod–Domar model
2. Neoclassical model (Solow)
3. Kaldor model
4. Endogenous Growth Model (AK)
5. Institutions, welfare and economic growth

# Harrod (1939) e Domar (1946)

- More attention to economic development
- Post-keynesians: Harrod and Domar
- Aggregate models: 1 good (homogeneous) (being as C and I).
- i.e. grain as C, grain as I (in order to avoid problems of value theory)
- No depreciation: ↑ I = ↑ stock K
- No Exp, no G, only real variables
- Long-term model where stock of K and productive capacity are never given data but are changing (→there is a function for I)
- H and D Same conclusions

# Domar model (1)

- $\Delta Y$= variation of effective Demand (= Income) from time t to time t + 1
- g = $\Delta Y$= Y2−Y1/Y1
- $\Delta I$ = variation of Investments from time t to time t + 1
- s = propensity to (marg./med.) to save

# Domar model (2)

- From the Keynesian multiplier theory we know that:

$$\Delta Y = (1/1-c)\ \Delta I$$

$$\Delta Y / \Delta I = (1/s)$$

$$\Delta Y = \left(\frac{1}{s}\right)\Delta I \qquad (*)$$

# Domar model (3)

- From our hp we obtain:

(No depreciation) **↑ I = ↑ stock K**

**I = ΔK** Given that **v = K / X**

( X=productive capacity)

Hp (reasonably): **Δ K / Δ X** constant through time

Remembering that I = ΔK →

$$\frac{I}{\Delta X} = v \;\rightarrow\; \frac{1}{\Delta X} = \frac{v}{I} \;\rightarrow$$

$$\Delta X = \frac{I}{v} \;(**)$$

# Domar model (4)

- In order to have equilibrium the development should be that the effective demand $\Delta Y$ = productive capacity $\Delta X$.
- →:

$$[\Delta Y = \Delta (C+I) = \Delta X] \rightarrow [\Delta Y = \Delta X]$$

Changing in $\Delta Y = \Delta X$ (*) and (**)

$$\underline{\boldsymbol{\Delta Y = (1/s)\, \Delta I}} = \underline{\boldsymbol{\Delta X = I / v}}$$

$$(1/s)\, \Delta I = I / v$$

$$\boldsymbol{\Delta I / I = s / v}$$

# Domar model (5)

$$\Delta I / I = s / v$$

- $g = \Delta I / I$, one can proof that $\Delta I / I = \Delta Y / Y$*

→

$$\underline{g = \Delta I / I = \Delta Y / Y = s / v}$$

**Note *: from $\Delta Y = (1/s)\, \Delta I$** divide by Y

**$\Delta Y/Y = (1/s)\, \Delta I/Y$** → $s=S/Y$ and $S=I$ in equilibrium

**$\Delta Y/Y = (Y/I)\, \Delta I/Y$**

**$\Delta Y/Y = (\underline{Y}/I)\, \Delta I/\underline{Y}$**

$$\Delta Y/Y = \Delta I/I = g = s/v$$

# Domar model (conclusions)

- A rate of growth g exists, that given s and K/X constant, maintains in every period effective demand and productive capacity equal ($\Delta Y = \Delta X$).
- Harrod will get to the same results and this rate of growth will be called **warranted** rate of growth (**$G^w$**)

# Harrod model (1)

- From the Keynesian function of S, we get, in time t:

**S = sY**

In order to have Supply = Demand we need:

**S = I** (ex ante)

Decisions about I are taken on the basis of the accelerator principle, i.e. they depend on expected variation of Y

$$I_t = v(Y_{t+1}^e - Y_t)$$

# Harrod model (2)

$$I_t = v(Y^e_{t+1} - Y_t)$$

v = K/Y

In order, for the expectations of investors to be verified, it is needed that:

**Yt+1=Yt+1 expected:**

**hence:**

$$I_t = v(Y_{t+1} - Y_t)$$

# Harrod model (3)

- But as we know:
  - $S = sY$,
  - $S = I$ and
  - $Yt+1-Yt= \Delta Y$   HENCE:

$$sY= v \Delta Y$$

$$\Delta Y /Y= s/v = Gg$$

**$G^w$ = warranted rate of growth =**

ratio between

Propensity to save **s** and capital/outcome **v**

# Harrod model (conclusions)

- $G^w = s/v$ as in the final equation of Domar: $g = s/v$. In Domar $v = K/X$ but for "normal" utilization of productive capacity $K/X = K/Y$

- $G^w$ is called warranted (not of equilibrium *tout court*) because although in here investors are satisfied and have the same rate of development as the one expected, this is an **instable equilibrium**

- $G^w$ **is the rate of growth at which I (and Y) have to grow in order to have equilibrium between Demand and Supply, and between production and productive capacity**

# $G^w$ = warranted rate of growth

- Given s e v one can get the $G^w$, the rate that "…if verified will guarantee to the investor the production in the same amount which will lead him to make orders and purchases that will maintain the same rate of development"
- $G^w$ guarantees that the firms' expectations concerning the demand in the following period are actually happening, hence, the aggregate demand will be equal to the productive capacity (supply)

# $G^w$ = warranted rate of growth (example)

- s= 20%
- v=5,
- Hence one need 5 unit of K in order to have 1 unit of Y :

$G^w = s/v = g$

$0.20/5 = 0.04 = 4\%$

With s and v constant the economy will growth at the same uniform rate of 4%

- $G^w = s/v_r = g = s/v_e \;=\; g{*}v_e = s = G^w{*}v_r =$

$G^w$ = warranted rate of growth
Vr= required K/Y
Ve = effective K/Y
g= effective rate of growth

# Warranted path of growth

- In order to have a warranted path of economic growth one needs that I increases through time in line with the productive capacity, so that the increased productive capacity finds its correspondence in the market and a stable growth of the aggregate demand

- At the same time to higher saving higher investments must correspond otherwise a fall in the aggregate demand will occur

# Instable equilibrium in Harrod the *knife edge*

- But growth is instable.

If for any reason the actual rate of growth get outside from the $G^w$ path, the market will not generate self-regulating equilibrium forces, on the contrary the actual path of growth will diverge more and more

If S>D, the excess will increase more (actual growth g will be lower and lower: $g < G^w$)

If D>S inflation will be higher and higher (actual growth will be higher than $G^w$ : $g > G^w$)

# Instable equilibrium in Harrod (1)

- Hp, g> $G^w$

(v required or wished/desiderata=5)

If $s/v_e > s/v_r$ (g effective > G warranted )

→ $v_e < v_r$ (4x1 instead 5x1) more efficient

- $K_e/Y < K_r/Y$ (K/Y effective < K/Y required)

The economy now needs less investment for the same outcome (4K not 5K)

If we consider the inverse Y/K, the productivity of K, Y increases with the same level of I (K)

Investors will be pushed to increase I more and more (Y will increase too). The divergence g> $G^w$ increases more and more

D > S → growing inflation

# Instable equilibrium in Harrod (2)

- Hp, $g<G^w$

(v required or wished/desiderata=5)

If $s/v_e < s/v_r$ (g effective < G warranted )

→ $v_e > v_r$ (6x1 instead 5x1) less efficient

- $K_e/Y > K_r/Y$ (K/Y effective > K/Y required)

The economy now needs more investment for the same outcome (6K not 5K)

If we consider the inverse Y/K, the productivity of K, Y decreases with the same level of I (K)

Investors will be pushed to decrease I more and more (Y will decrease too). The divergence $g< G^w$ increases more and more D < S → growing unemployment

# Examples1

Stable growth, g=4%, $G^w$ =4%

| T | Income (bln) Yt=Yt-1*1,04 | Wished capital stock K*t=5Yt | I=S=It =0.20Yt | Effective capital stock Kt=Kt-1+It-1 |
|---|---|---|---|---|
| 0 | 100 | 500 | 20 | 500 |
| 1 | 104 | 520 | 20,8 | 520 |
| 2 | 108,16 | 540,8 | 21,6 | 540,8 |
| 3 | 112,48 | 562,43 | 22,49 | 562,4 |
| 4 | 116,99 | 584,9 | 23,4 | 584,9 |

Time 1 Investors invest 4% of investments (i.e 800mln→20,8bln), → Y = (with multiplier =5 → 5*0.8=4. Y= 104 (or 100*1.04=104) K*=(5Y)=(5*104)=520 = K effective (500+20)

In time 2: I=4%= 832mln (→21.6bln)
5*0.832=4.16. Y=104+4.16=108.16
K*=540.8= K effective (520+20.8)

# Examples 2

Unstable growth, g=5%, $G^w$ =4%

| T | Income (bln) Yt=Yt-1*1,04 | Wished capital stock K*t=5Yt | I=S=It =0.20Yt | Effective capital stock Kt=Kt-1+It-1 |
|---|---|---|---|---|
| 0 | 100 | 500 | 20 | 500 |
| 1 | 105 | 525 | 21 | 520 |
| 2 | 110,25 | 551,25 | 22,05 | 541 |
| 3 | 115,76 | 578,81 | 23,15 | 563,05 |
| 4 | | | | |

But: If Investors for some reasons invest more that 4% (i.e 5%), for instance 21 bln instead of 20,8 → Y=105 instead of 104
And since I=S, if I=5 with the multipliear (1/s=5) equal to 5, Y=105
K*=525 instead of 520. A paradox will emerge: since investors invested more, have now K<K* → D>S→ growing inflation. In time 3 they will invest even more since they think it is necessary to increase the capital stock…(PMK↑ → I↑)

# Examples 3

Unstable growth, g=3%, $G^w$ =4%

| T | Income (bln) Yt=Yt-1*1,04 | Wished capital stock K*t=5Yt | I=S=It =0.20Yt | Effective capital stock Kt=Kt-1+It-1 |
|---|---|---|---|---|
| 0 | 100 | 500 | 20 | 500 |
| 1 | 103 | 515 | 20.6 | 520 |
| 2 | 106,09 | 530,45 | 21,218 | 540,6 |
| 3 | | | | |
| | | | | |

But: If Investors for some reasons invest less that 4% (i.e 3%), for instance 20,6 instead of 21 bln → Y=103 instead of 104
And since I=S, if I=5 with the multipliear (1/s=5) equal to 5, Y=103
K*=515 (515*3=515) instead of 520. Another paradox will emerge: since investors invested less, have now K*<K → S>D→ unemployment and recession.
They will invest even less in time 3 because they think that is necessary to reduce the productive Capacity, and instead they will even less growth

(PMK ↓ → ↓I)

# Natural rate of growth, $G^n$

- $G^n$= max rate of growth possible (allowed for by population growth n, capital accumulation, technical progress)

$$\frac{\Delta Y}{Y} = \frac{\Delta N}{N} + \frac{\Delta \pi}{\pi} + \frac{\Delta N}{N}\frac{\Delta \pi}{\pi}$$

$$G^n = n + \lambda$$

# $G^n = G^w$ ?

$$G^n = n + \lambda = \mathbf{s/v} = G^w$$

- This only possible by chance…
- It does not exist any mechanism self-regulating that guarantee that $G^n$ will be equal to $G^w$, so that the system will develop in equilibrium and full employment

# Case 1 : $G^n > G^w$

- $G^n = 5\%$ ($n = 2\%$, $\lambda = 3\%$)
- $G^w = 4\%$

Hp 1) $g > G^w <= G^n$. $g = 5\%$ → instability with growing inflation

Hp 2 ) $g = G^w < G^n$. $g = 4\%$ → instability with growing unemployment

FE, $\lambda = 3\%$, with $g = 4\%$ is needed $n = 1\%$

**Structural unemployment** due to scarcity of capital (and not to lack of demand… )

We are in fact on the guaranteed path ($g = G^w = 4\%$) (the demand absorbs all productive capacity) but K does not growth enough to guarantee full employment!)

# Case 2 : $G^n < G^w$

- $G^w = g\ (7\%) > G^n\ 5\%$
- $G^n = 5\%\ (n=2\%, \lambda = 3\%)$

This is possible only starting from a situation of already unemployment, and $g>G^n$ is possible as far as unemployment reserves allow for that.

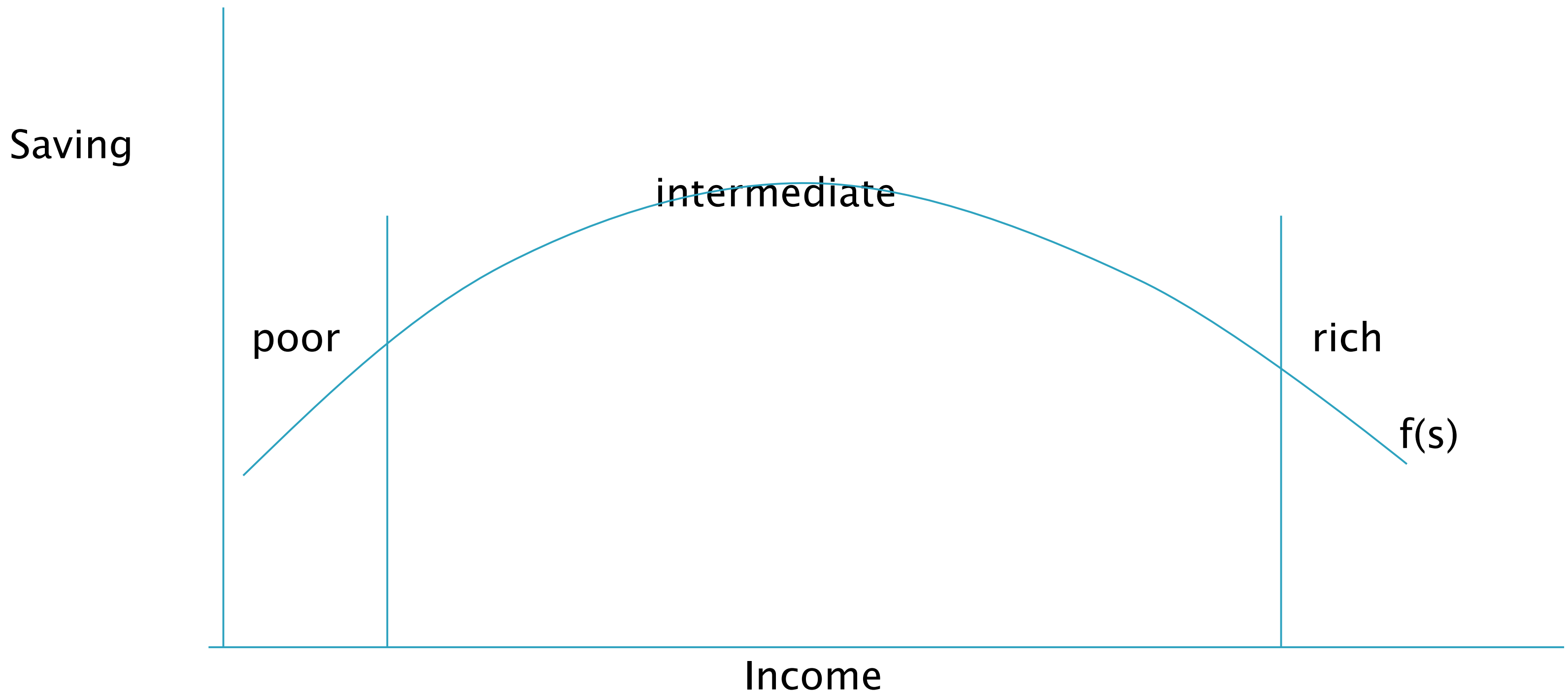
This is possible until when unemployment disappears. Afterward g needs to go towards Gn.

Given that $G^n < G^w \rightarrow g < G^w$. Hence we will get instability with growing unemployment (keynesian) due to lack of demand ($K/Y > Kd/Y \rightarrow$ this leads to invest even less…) $\rightarrow g << G^w \rightarrow$ growing unemployment

# Conclusions

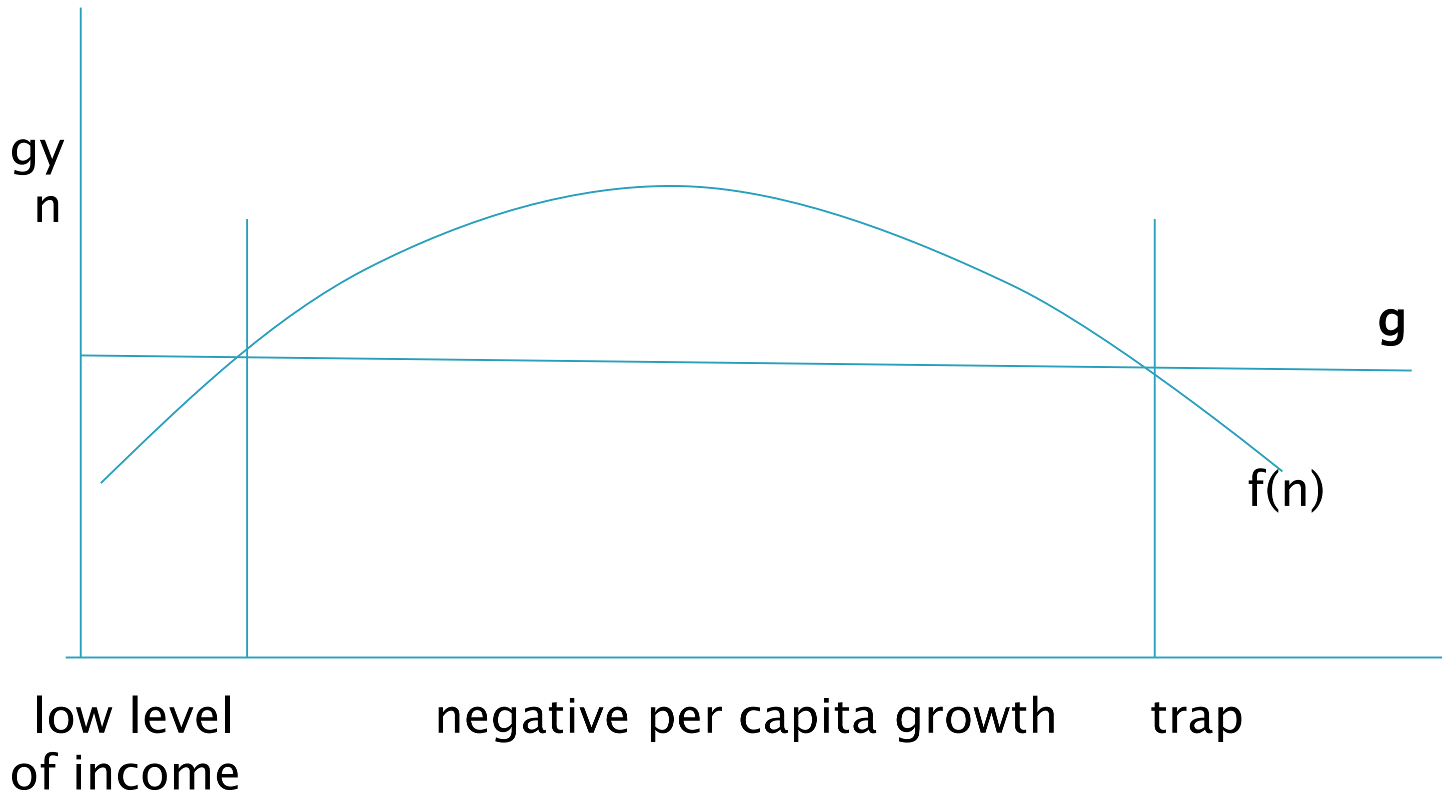
- G in LT → constant
- + s → + g
- Prouctivity of K (Y/K) → g (efficiency)
- Depreciation (–) → g
- S is endogenous because depends on income
- n is "endogenous" too, because depends on income

# Saving function (endogenous transition), H-D→ policy role




Rich countries growth faster because higher s. Contrary poor coutries. Intermediate countries grow the fastest (high s, lower C)

# Demographic transition (endogenous path) H-D → policy role: Push Y after the trap, with more Inv ( g towards up, or n towardx down)




**n is low for low level of Ypc.**
**n is higher for higher level of Ypc (natality constant, mortality ↓).**
**In the intermidiate phase of higher n, per capita grotwh ↓**
**Later: natality ↓, when Y pc ↑**